Saturated and unsaturated fluid flow in a centrifuge

Gary F. Goforth
South Florida Water Management, West Palm Beach, Florida

F. C. Townsend and D. Bloomquist
University of Florida, Gainesville, Florida

ABSTRACT: The feasibility of using centrifuge techniques to predict the movement of fluid through saturated and unsaturated soil was evaluated by a laboratory testing program and computer simulations. Fluid kinematics within a soil undergoing radial acceleration were derived. Saturated permeability values for a sand and a sand/clay agreed for both centrifuge and 1-g bench tests. Advection of fluids through some unsaturated soils can be governed by suction gradients, and for these cases centrifugal modeling is inappropriate. However, centrifugation may be feasible for developing hydraulic conductivity-volumetric water content relationships.

1 INTRODUCTION

The assessment of local and regional impacts on groundwater resources due to leachate of wastes from confined disposal areas and accidental spills necessitates the prediction of contaminant migration. In general, either a physical or numerical model can be applied to depict the mass transport phenomena. The objective of this study was to assess the technical feasibility of using centrifugal modeling for determining the migration rate characteristics of leachate wastes in soils for saturated and unsaturated conditions. Accordingly, the scope of work involved: (a) design, fabrication and analysis of permeameters in 1-g and multiple-g environments, (b) derivation of fluid flow equations and (c) hydraulic conductivity tests on saturated and unsaturated sand and sand/clay soil samples.

2 BACKGROUND

2.1 Centrifugal fluid mechanics

Essential to the design and analysis of a permeameter subjected to radial accelerations is an understanding of fluid flow through conduits and saturated porous media. Figure 1 represents a mass of fluid in a tube, with motion parallel to the radial acceleration, the acceleration, a_r, acting on the mass is a function of the radius, r, expressed as $a_r = r\omega^2$, where ω = angular velocity (rad/t). Summing the forces acting on the element and incorporating anticipated force, yields

$$PdA - (P+dP)dA - dF_s + \rho a_r dAdr = \rho drdAdv/dt \quad \text{. . . . . eqn (1)}$$

where P = pressure acting on the surface, F_s = total shear forces, and dv/dt = acceleration. Simplifying and replacing dr/dt with the fluid velocity, v, and dF_s with an energy loss term, dH_L, yields:

$$\left(-\frac{dP}{\rho}\right) - dH_L + \omega^2 r dr = Vdv \quad \text{. . eqn (2)}$$

For an incompressible fluid, eqn (2) is integrated across the element to yield

$$(P_2-P_1)/\rho + H_L - \omega^2(r_2^2-r_1^2)/2 + (V_2^2-V_1^2)/2 = 0 \quad \text{. . . . . eqn (3)}$$

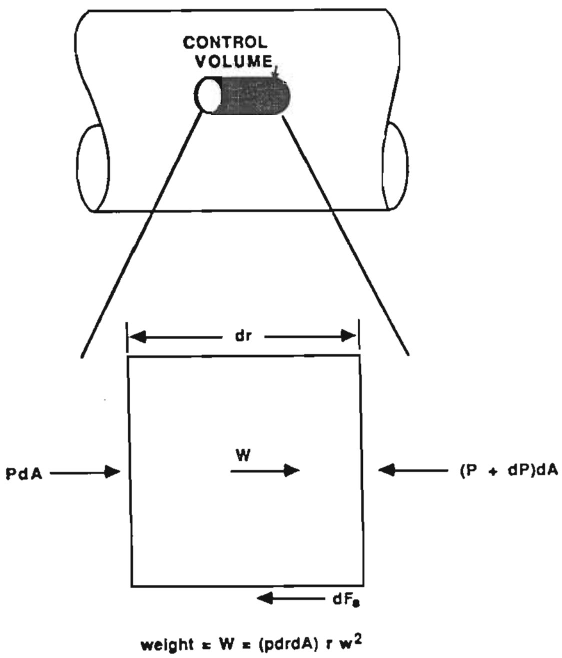

Figure 1 Forces Acting on a Fluid Volume in a Centrifuge

Equation 3 is the centrifuge equivalent of the Bernoulli equation rewritten as:

$$\left(\frac{V^2}{2} + \frac{P}{\rho} - \omega^2 r^2/2\right)_1 = (V^2/2 + P/\rho - \omega^2 r^2/2)_2 + H_L \qquad \text{eqn (4)}$$

For flow through porous media, the velocity component of the hydraulic potential is negligible compared to pressure and elevation terms, thus Darcy's law within a centrifuge soil sample can be expressed as:

$$q = -k\frac{d}{dr}(P/\rho - \omega^2 r^2/2) \quad . . \quad \text{eqn (5)}$$

It is important to note from eqn 5, that the increase in discharge is directly proportional to the acceleration level only if the pressure gradient is zero. Furthermore, an interchangeable relationship between a pressure differential or increase in centrifugal acceleration exists to create the hydraulic gradient required for flow. That is to say, pressure gradients across a saturated sample can be reproduced by acceleration levels and vice-versa.

2.2 Advection and dimensional analysis

Advection, the flux of fluid through a soil volume, is proportional to the existing potential gradient, as stated by Darcy's law, written as: q = -k(dH/dz) where q = volume of fluid passing through a unit area in a unit time (L/T). The gradient of the total hydraulic potential (dH/dz) provides the driving force for fluid movement in soils. An apparent discrepancy concerning the scaling of hydraulic conductivity (k) arises based upon an inconsistent definition of the total potential energy (H).

When the total potential energy is expressed as energy (F·L) per unit weight (F), defined as hydraulic potential, or head, then head has the dimension of length (L). Consequently, the potential gradient is dimensionless and scales as unity (1), while hydraulic conductivity (k) has the units of (L/T) and will scale as 1/N, where N is the acceleration (or scaling) ratio of model to prototype. (Remembering that for fluid flow, time models as N and force models as N^2, Scott and Morgan, 1977.)

If the potential energy is expressed as energy (F·L) per unit volume (L^3), defined as pressure potential, then its dimension is (F/L^2). Consequently, the potential gradient has dimensions (F/L^3) and scales as $^1/N$, while hydraulic conductivity has the units of (L^4/TF) and scales as unity (1).

Darcy originally used the dimension of length for hydraulic potential, and thus the dimensions of hydraulic conductivity (k) were L/T. The influence of acceleration can be separated by employing intrinsic permeability (K) with dimensions (L^2) and expressed as $k = Kg/\mu$, where k = hydraulic conductivity (L/T), g = gravity (L/T^2) and μ = kinematic viscosity (L^2/T).

3 TESTING PROGRAM

The University of Florida geotechnical centrifuge has a 1-m radius and can accelerate 25 kg to 85 g's. A photo electric pick-off and flash delay facilitate visual observation and photographic recording. Two hydraulic slip rings supply fluid to the interior, while 32 electrical slip rings are available for signal monitoring.

3.1 Soils

Two soil preparations were utilized; (1) a fine grained silica sand (SP), D_{50} = 0.2mm, and (2) a mixture of 80% sand and 20% kaolinite (by weight).

3.2 Permeameter

The flexible wall permeameter (10-cm diameter) used for the centrifuge and bench tests is presented schematically in Figure 2. It has the capability of vacuum deairing and back-pressure saturation and is completely self-contained to maintain balance while in flight. The unit consists of a 1.25-cm thick, 11.43-cm inside diameter acrylic cylinder separated by 2.54-cm thick acrylic plates. The entire apparatus was unified by six 0.95-cm diameter steel rods, while O-rings between the individual elements provided high pressure seals. Flow between the reservoirs and the soil sample was controlled by a three-way solenoid valve. Energy losses were monitored via differential pressure transducers located at the influent and effluent reservoirs.

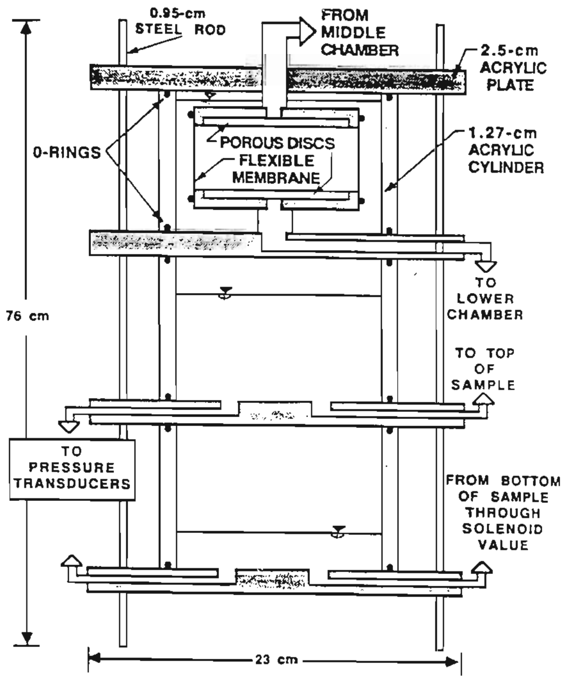

Figure 2 Schematic of Saturated Hydraulic Conductivity Test Apparatus

3.3 Testing procedures

Both the bench and centrifuge permeability samples were saturated by flow-through back pressure saturation and verified by "B" value checks. A range of gradients was established during the saturated permeability testing in anticipation of determining the threshold Reynolds number for Darcy's law validity. After the desired initial pressure boundary conditions were established and fluid levels in the reservoirs recorded, the solenoid valve was opened and flow through the sample commenced. At the end of the test, the solenoid valve was closed and elapsed time and fluid levels were recorded.

The centrifuge tests were conducted on the soil specimen immediately following the 1-g bench tests. The pressure transducers were recalibrated before each centrifuge test to compensate for line noise in the electrical slip rings. During the centrifuge tests, pressures in the sample and fluid reservoirs were controlled by regulators external to the centrifuge, which supplied air through the hydraulic slip rings.

3.4 Data analysis

Analysis of the test results required derivation of the appropriate flow equations based upon accelerations and boundary conditions. The variable head equation for the 1-g bench tests was determined (Goforth, 1987) to be

$$k = \frac{aL}{2At} \ln (h_i/h_f) \quad . . . . . \text{ eqn (6)}$$

where: a = cross-sectional area of the influent line, L and A = sample length and area, and t = test duration;

$$h_i = \frac{P_M - P_L}{\rho - g} + (Z_{MO} - Z_{LO}) \quad . . \text{ eqn (7)}$$

and $h_f = h_i + 2h$, where h = rise in the effluent reservoir surface, and both reservoirs are assumed to have equal areas (see Figure 3). The units of k are length per time. The variable head equation for the centrifuge tests with k in units of time is:

$$k = \frac{aL}{At\,h_o} \ln h_1/h_2 \quad . . . . . . \text{ eqn (8)}$$

where: $h_o = \omega^2(r_{LO} + r_{MO})$, r_{LO}, r_{MO} = initial radii of the water surfaces; and

$$h_1 = \frac{P_L - P_M}{\rho} + \frac{\omega^2}{2}(r_{MO}^2 - r_{LO}^2),$$

and $h_2 = h_1 + h_o * h$, with h = increase in radius of the upper fluid surface (see Figure 3).

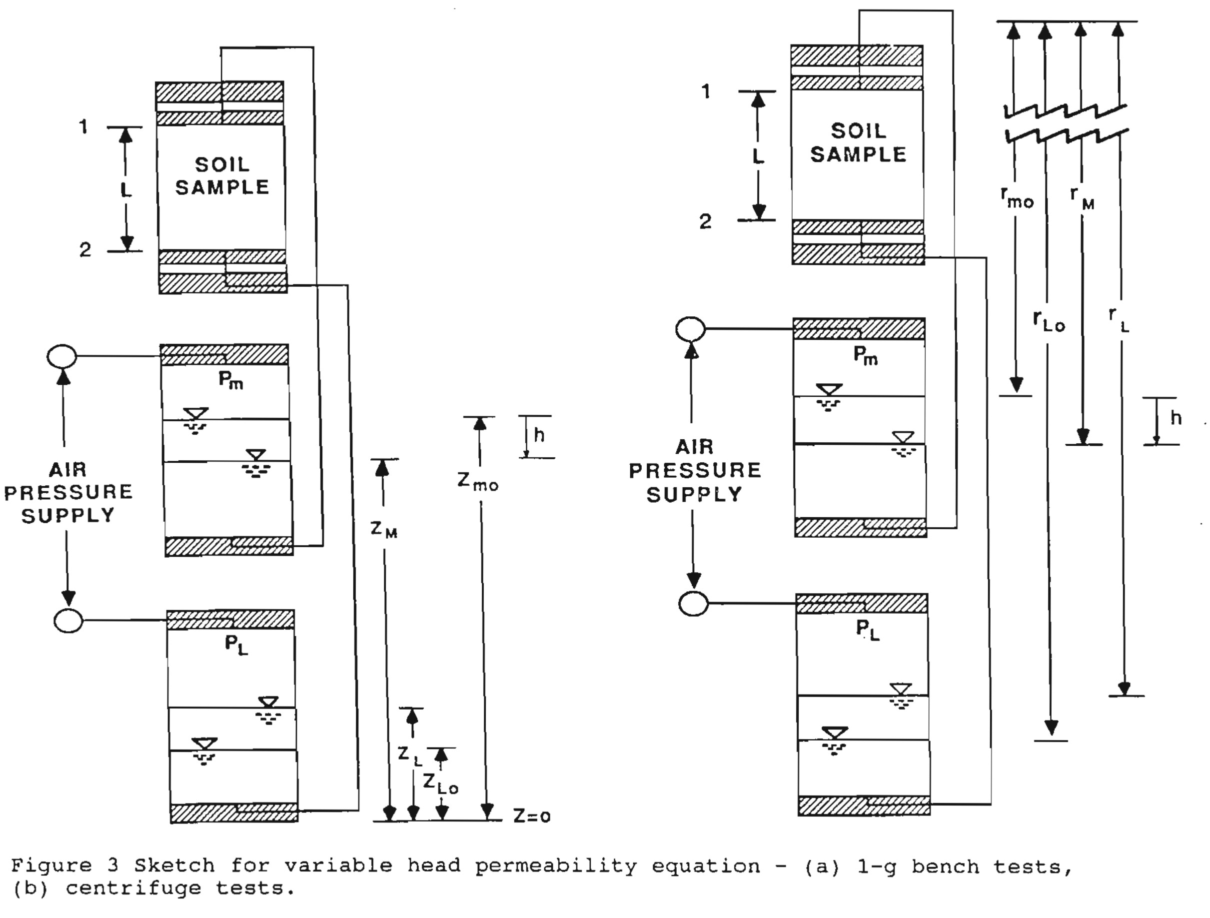

Figure 3 Sketch for variable head permeability equation - (a) 1-g bench tests, (b) centrifuge tests.

4 SATURATED TESTS RESULTS

Departure from Darcy's law was observed in both the 1-g and multiple-g tests with sand. As Figure 4 demonstrates, the deviation from Darcy's law was reproduced in the centrifuge at accelerations of 14.7 and 24.4 g's. The greater scatter observed in the centrifuge results is attributed to observed fluctuations caused by sloshing in the reservoir surfaces. Somewhat constant values of k were determined below a gradient of around ten, corresponding to a soil Reynolds number of approximately 0.2. This value is almost an order of magnitude smaller than the reported limits of Re between one and ten (Bear, 1979). Figure 5 compares the results obtained in the centrifuge at 19.3 g's with those determined on the bench for the sand/clay soil. Initial gradients between 90 and 200 were established across the 4.8-cm long samples during the tests.

Estimates of the intrinsic permeability of water through a sand/clay sample obtained in the centrifuge at 160 and 180 RPM (19.3 and 24.4 g's, respectively) are compared in Figure 6. The lower estimates observed at the higher acceleration level suggest that the greater confining pressures, and consequently, greater effective stresses on the sample, reduced the rate at which water moves through the soil pores.

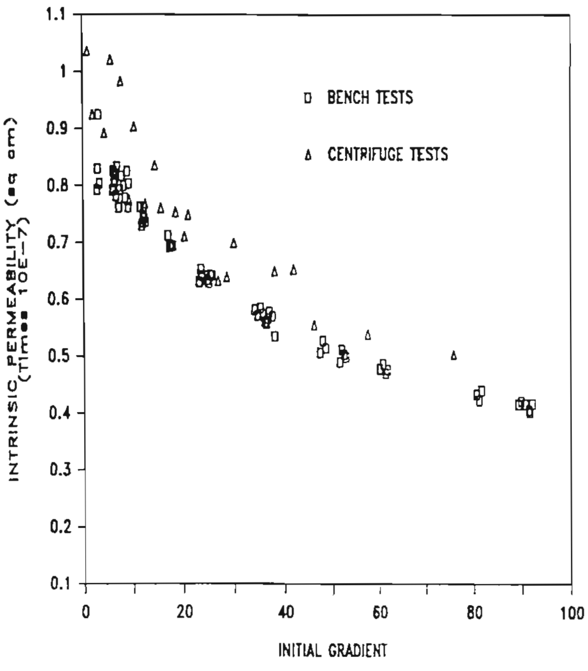

Figure 4 Comparison of centrifuge and bench permeability test results on sand for 1, 14.7, and 24.4 g's

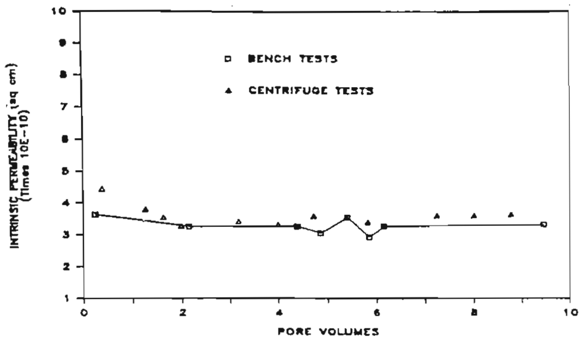

Figure 5 Comparison of centrifuge and bench permeability test results on sand/clay for 1, and 19.3 g's

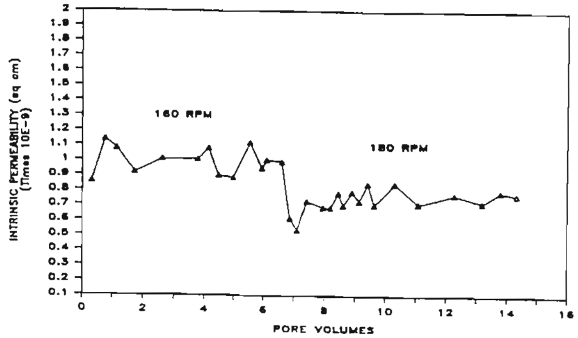

Figure 6 Comparison of sand/clay permeability at 19.3 and 24.4 g's

5 FLOW IN UNSATURATED MEDIA

5.1 General

As in the case in saturated soil, hydraulic potential gradients determine the flow conditions in unsaturated soil. However, unlike the positive pressures acting on the pore fluid in saturated media, pressures less than atmospheric exist within unsaturated soils. These negative pressures, termed soil suction, are a hysteretic function of the soil moisture content and at the same moisture content depend upon whether arrived at by wetting or drying cycles. Generally, fluid flux in soils is dominated by suction gradients, which can be 10 to 1000 times greater than the gradient due to gravity (Hillel, 1982). In a uniformly dry soil, where suction gradients dominate water movement below an influent source will occur in a radial pattern (see Figure 7), demonstrating the negligible influence of gravity.

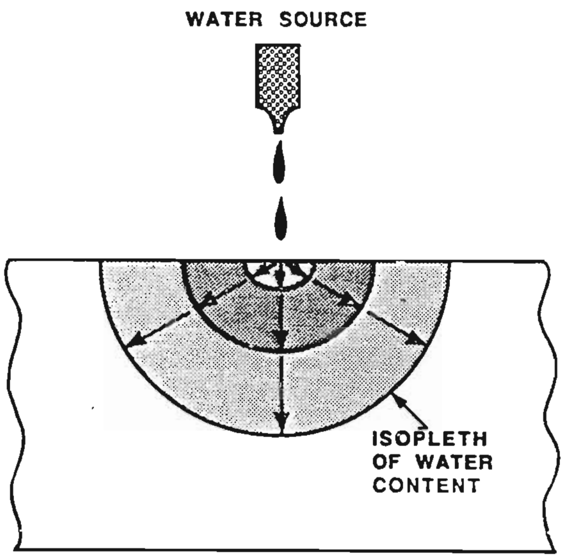

Figure 7 Radial moisture movement in a uniformly dry soil

5.2 Unsaturated hydraulic conductivity tests

Although there appears to be no advantage of modeling unsaturated flow in a gravity accelerated environment using soils dominated by suction gradients, centrifugation may offer benefits in determining unsaturated hydraulic conductivity-moisture content relationships. One laboratory test method, the instantaneous profile method (IPM), was selected as the best candidate test amenable to centrifugation. This transient flow test for measuring unsaturated hydraulic conductivities has a time advantage over steady-state methods in that k values over a range of moisture contents are obtained during a single test. The IPM test involves monitoring the change in soil suction with time as a soil column drains from an initially saturated condition, or during imbibition by a dry soil (Green, 1983 & Hillel 1982). Obviously, during initial drainage the gravitational potential gradient exceeds the soil suction gradient (Dane 1983).

5.3 Numerical simulation of unsaturated flow

A fully implicit finite difference computer program was developed to evaluate the influence of elevated acceleration levels on soil movement in unsaturated soils. The model incorporated the centrifuge version of Darcy's law (eqn 5) into Richard's 1-D continuity equation: $d\Theta/dt = -dq/dt$, where $d\Theta/dt$ is the time rate of change in volumetric water content. A nonlinear exponential function was used to describe the hydraulic conductivity-volumetric water content relationship of a hypothetical sand sample (Van Genuchten, 1979). Figure 8 compares the simulated benefits of centrifugal tests of an initially saturated sample drained by gravity over 1-g bench tests for the IPM and shows two obvious advantages: (1) centrifugation covers a much wider range of soil moisture and suction; and, (2) the testing time is reduced. An additional advantage of centrifugal testing is the rapid determination of moisture retention characteristics of slow draining soils by spinning saturated samples until drainage ceases and subsequently determining the moisture content profile.

5.4 Unsaturated soil laboratory tests

Figure 9 presents a schematic of the apparatus used for IPM tests. The 2 solenoid values were used to saturate and drain the soil sample of sand while in flight. Pressure/suction changes throughout the test were monitored by 5 Druck transducers (0.05v/cm water). Unfortunately due to problems in maintaining good contact between the sand and transducers, system saturation and our ability to measure suctions, with these transducers, we were unsuccessful in obtaining reasonable repeatable data for most tests. Perhaps a more impermeable soil and better saturation techniques, and different suction measuring devices would lead to better data.

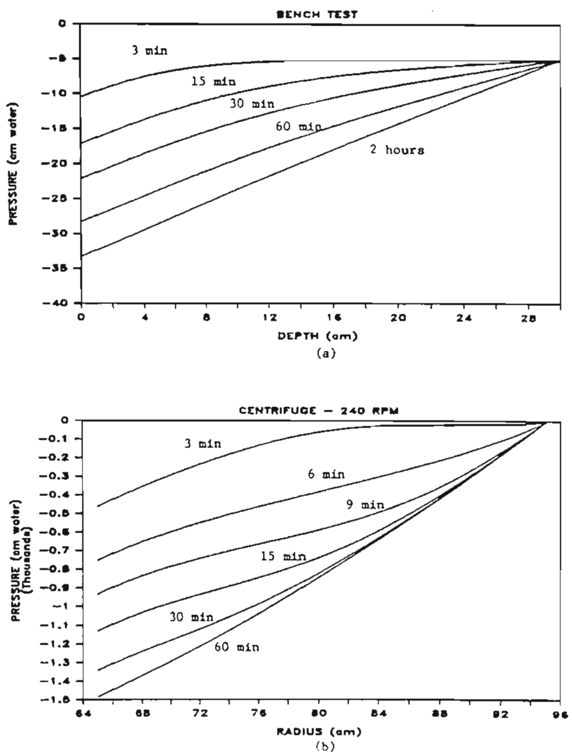

Figure 8 Comparison of pressure profiles (a) bench simulation; (b) centrifuge simulations

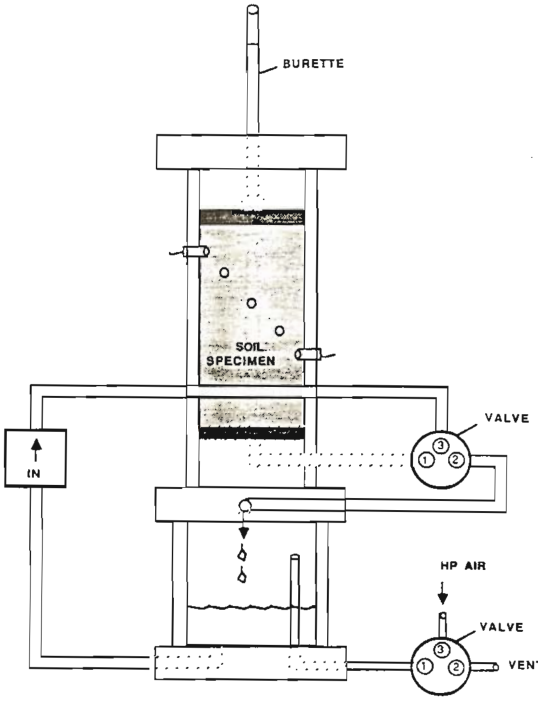

Figure 9 Schematic of unsaturated flow apparatus

6 CONCLUSIONS

1. For saturated soils, equations of fluid motion in a porous media within a centrifuge were derived. Excellent agreement was obtained for intrinsic permeabilities measured by 1-g bench and centrifugal model tests.
2. The centrifuge technique for determining saturated hydraulic conductivity does not offer any advantage over similar 1-g bench tests. Equations of motion indicate an interchangeable relationship between pressure differential and centrifugal acceleration.
3. Sloshing of the reservoir surface during centrifugal model tests resulted in less accuracy of conductivity values than from 1-g bench tests. A deviation from Darcy's law was observed in sand samples at a Reynolds number greater than 0.2.
4. Employing a centrifuge to model percolation of waste leachate through an unsaturated soil profile may not be feasible, if soil suction gradients dominate water movement in an unsaturated soil.
5. Computer simulation suggests that conducting the instantaneous profile method (IPM) test for determining unsaturated hydraulic conductivity may be more advantageous timewise in a centrifuge than 1-g bench tests. However, our test equipment and methods were unable to verify this simulation.

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